MATHEMATICAL MODEL OF LEAD TRANSFER IN HUMAN BODY

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To my beloved mother and father

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Abstract

A mathematical model of lead transfer in human body is developed by using compartmental model based on previous studies and formulated in mathematical terms to calculate and predict the level of lead in human body. The Batschelet's three-compartmental model being used in this study is adopted from the Rabinowitz *et al.* (1976). The experimental data adopted in this study is drawn from the published researches to test the mathematical model for solving the real biological system illustrated by the Batschelet's three-compartmental model and to ascertain the mathematical model fit the experimental data. Some examples of a real biological system of lead transfer in human body are studied such as the amount of lead and the equilibrium state of lead level stored in several major parts of human body. A mathematical model derived from the compartmental model which involves first-order ordinary differential equations is solved analytically by using a mathematical technique which utilize some basic concepts from the algebra of matrices. The mathematical general solution obtained is employed by using Microsoft Excel program to produce tables and plot graphs.

Abstrak

Model bermatematik bagi peralihan jisim plumbum dalam tubuh manusia dibangunkan dengan menggunakan model berpetak berdasarkan kajian lepas dan dirumus dalam sebutan bermatematik untuk menghitung dan meramal aras jisim plumbum dalam tubuh manusia. Model berpetak-tiga Batschelet dalam kajian ini mengguna pakai model yang dipelopori oleh Rabinowitz *et al.* (1976). Data kajian yang digunakan pula diperoleh daripada kajian lepas untuk diuji pada model bermatematik tersebut bagi menyelesaikan sistem biologi sebenar yang diterjemahkan oleh model berpetak-tiga Batschele dan untuk menentusahkan sama ada model bermatematik tersebut bersesuaian dengan data kajian. Beberapa contoh sistem biologi sebenar bagi peralihan jisim plumbum dalam tubuh manusia yang boleh dikaji ialah amaun plumbum dan takat keseimbangan aras plumbum tersimpan dalam beberapa bahagian utama tubuh manusia. Model bermatematik yang diterbitkan daripada model berpetak tersebut mengandungi persamaan-persamaan terbitan biasa darjah-pertama diselesaikan secara analitik dengan teknik bermatematik yang melibatkan beberapa konsep asas matriks aljabar. Penyelesaian umum bermatematik yang diperoleh digembleng dengan mengguna pakai Microsoft Excel untuk menghasilkan jadual dan memplot graf.

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D	-	lead intake
$x_i(t)$	-	amount of lead in compartment i at time t
$x_{i'}(t)$	-	lead transfer rate in compartment i at time t
α	-	rate of lead enters the lungs
β	-	rate of lead enters the digestive system
y_{ij}	-	rate of lead transfer from compartment i to j

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CHAPTER 1

INTRODUCTION

1.1 Background of The Problem

Lead is a toxic metal and it is closely related to human life. It is used in some products that could be found around us such as paint and cosmetic (Moore et al., 1980, and Paoliello and Capitani, 2007). It is also a natural occurring chemical element contained in soil. It is estimated that the level of lead in soil is in between 50 ppm and 400 ppm (US EPA, August 3, 2012). In our current societies, the environment has been polluted with lead from the sophisticated industry such as in battery and paint manufacturing. Moreover activities such as mining, smelting and refining also resulted in the increase of substantial amount of lead level in the environment (Maja, 1996 and Niisoe *et al*., 2011).

Due to the concern and awareness of the hazardous of lead in the environment to human health, some efforts have been taken aiming to reduce the level of human exposure to lead, such as the banned of the use leaded gasoline, closing the lead refinement and lead mining plants (US EPA, August 3, 2012, Lemos and Carvalho, 2010, and Paoliello and Capitani, 2007). Basically, human exposure to lead can happen through two major pathways, the gastrointestinal tract (GIT) and the respiratory tract (Niisoe *et al*., 2011). The GIT intake depends on the

contamination level of food and water while the respiratory tract depends on the air concentration of contaminant (Niisoe *et al*., 2011).

The human exposure to lead could be studied through the lead level in human body. Lead level in human body is measured by using spectrometric methods to detect the presence of lead in human body parts. The experimental data obtained from a previous study could be used to study the lead level in human body through a mathematical model. The mathematical model which is derived from a compartmental model based on previous studies reflect the real physical system of lead transfer in human body. The mathematical model can be solved using mathematical technique. This dissertation compares and analyzes the model with different perspectives.

1.2 Statement of the Problem

As mentioned earlier in the previous section, lead level in human body is studied by using a mathematical model. The mathematical model is derived from the Batschelet's three-compartmental model adopted from the Rabinowitz *et al.* (1976)'s compartmental model which reflect the kinetics of lead in human body. The parameters are estimated from the field data drawn from the study made by Rabinowitz *et al.* (1973). However, most of the solutions of the studies on lead level in human body made by previous researchers such as Batschelet *et al.* (1979) and Steyn *et al.* (2008) used numerical solutions approach to solve the mathematical model. Thus, this dissertation used an analytical approach to solve the mathematical model of lead transfer in human body and to show that results produced by analytical approach correspond almost perfectly with those of a numerical approach.

1.3 Objectives of the Study

The objectives of this study are:

(i) to conceptualize compartmental model for lead transfer in the human body
(ii) to formulate mathematical model based from the compartmental model for lead in the human body
(iii) to solve the mathematical model analytically
(iv) to discuss the result obtained by comparing with published researches

1.4 Scope of the Study

The compartmental system for lead in the human body will be interpreted into mathematical model involving differential equations in which each equation represents the lead concentration in several major parts of human body. The mathematical model was solved theoretically using the data drawn from Rabinowitz *et al.* (1973/76) studies.

1.5 Significance of the Study

The significance of the study can be seen in two different perspectives, theoretically and practically. Theoretically, the study consists of multidisciplines, the mathematical and biological disciplines at which the real biological system is interpreted in mathematical model and formula. Therefore the study is deemed to

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